Second 0+ state of unbound $^{12}$O: Scaling of mirror asymmetry

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The unbound $^{12}$O nucleus was studied via the two-neutron transfer ($p,t$) reaction in inverse kinematics using a radioactive $^{14}$O beam at 51 MeV/u. Excitation energy spectra and differential cross sections were deduced by the missing mass method using MUST2 telescopes. We achieved much higher statistics compared to the previous experiments of $^{12}$O, which allowed accurate determination of resonance energy and unambiguous spin and parity assignment. The $^{12}$O resonance previously reported using the same reaction was confirmed at an excitation energy of $1.62\pm0.03$ stat. $\pm0.10$ syst. MeV and assigned spin and parity of $0^+$ from a distorted-wave Born approximation analysis of the differential cross sections. Mirror symmetry of $^{12}$O with respect to its neutron-rich partner $^{12}$Be is discussed from the energy difference of the second $0^+$ states. In addition, from systematics of known $0^+$ states, a distinct correlation is revealed between the mirror energy difference and the binding energy after carrying out a scaling with the mass and the charge. We show that the mirror energy difference of the observed $0^+$ state of $^{12}$O is highly deviated from the systematic trend of deeply bound nuclei and in line with the scaling relation found for weakly bound nuclei with a substantial $2s_1/2$ component. The importance of the scaling of mirror asymmetry is discussed in the context of ab initio calculations near the drip lines and universality of few-body quantum systems.

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I. INTRODUCTION

Light nuclei near the drip lines provide unique access to nucleons weakly bound at the Fermi surface. It is widely accepted that these nucleons, when filling orbitals with low angular momenta ($L$), reach far beyond the classical turning point of a binding potential via tunneling effects, generating various exotic phenomena that are unusual in stable nuclei, such as neutron halos [1,2], coupling to the continuum [3,4], dineutron correlations [5–7], or universal three-body states [8–11].

Experimental data that stress the importance of finite binding effects are accumulating. Recent studies [12,13] presented a unique finding that some observables of weakly bound states show a simple and smooth evolution as a function of binding energies, as if details of short-range interactions are overshadowed by geometrical effects of finite binding. For instance, Riisager et al. revealed that the sizes of halo states are inversely related to the binding energies, and their relation only depends on angular momenta $L$ of valence nucleons after scaling with the reference length and mass [9,12]. Hoffman et al. found that energies of $2s_1/2$ states relative to $1d_{5/2}$ states smoothly decrease toward the drip line in light nuclei with a neutron number of $N = 5$ to 10. These systematic evolutions point to crucial roles of finite binding in universal scaling of halo state properties [9,12] or in describing major changes in shell structure [13].

Accurate treatment of small $L$ orbitals near thresholds constitutes a major challenge in modern structure calculations [5,6,8–11,14–17]. As these orbitals require much larger coordinate space than others, large-scale calculations based on the

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shell model with residual interactions [18,19] or the \textit{ab initio} approach using realistic nuclear forces [20–23] are rendered highly difficult. Special techniques such as the Gamow shell model using complex eigenstates [15], an effective interaction from the monopole-based-universal interaction in Woods-Saxon bases [16], or the coupled-cluster method [17] have been developed to describe nuclei near and beyond the drip lines. Another group of theories are based on simplified models often assuming a cluster of an inert core and a few valence nucleons for the sake of enabling accurate and explicit treatment of finite potentials [5,6,8–11]. For better treatment and understanding of finite binding effects, it is important to track evolution of physical observables sensitive to these effects toward the drip lines.

In this paper, we report on the level scheme of the unbound and proton-rich $^{12}$O nucleus and discuss how weak binding effects characterize the evolution of mirror symmetry breaking. Asymmetry in level energies between mirror nuclei is another observable sensitive to weakly bound orbitals with small $L$. The asymmetry for $s_{1/2}$ states is well known as the so-called Thomas-Ehrman shift [24,25], in which excitation energies of $s_{1/2}$ states in proton-rich nuclei are much lower than in neutron-rich mirror nuclei. This is due to the unusually lower Coulomb energy of an $s$-wave proton with a broader wave function that overlaps less with the rest of the nucleus compared to other orbitals with higher angular momenta. This mechanism makes the difference in energies of mirror states, or mirror energy difference, a very sensitive probe of the spatial structure of orbitals with low angular momenta.

Mirror symmetry of $^{12}$O, the lightest oxygen nucleus ever found, will be revealing, as it is located beyond the proton drip line and unbound for two-proton emission by 1.638(24) MeV [26]. The $0^+_1$ state predicted near 2 MeV [27,28], and thus more unstable than the ground $0^+$ state, is of special interest. In the mirror nucleus $^{12}$Be, the $0^+_2$ state was found at 2.25 MeV by in-beam $\gamma$-ray spectroscopy [29,30] and later confirmed by the charge exchange reaction [31]. The valence neutrons of this $0^+_2$ state are thought to occupy the $2s_{1/2}$ orbital as well as the $1p$ orbitals [29–31] due to the disappearance of the shell closure at $N=8$ in $^{12}$Be [32,33]. While the ground state of $^{12}$O was already observed in the 1970s [34–36], excited states had been elusive ever since, until a resonance was identified at 1.8(4) MeV in our previous study of the $^{14}$O($p,t$)$^{12}$O reaction by the missing mass method [37,38]. Its much lower excitation energy ($E_x$) compared to $^{14,16}$O undoubtedly evidences the disappearance of the shell closure at the proton number $Z=8$. However, the statistics were still limited and led to a large error of $E_x$ and ambiguity of the spin and parity ($J^\pi$), with a tentative assignment of $0^+$ or $2^+$. Another observation of a resonance at 1.968(52) MeV later came from the one-neutron knockout reaction of $^{13}$O [39]. The statistical quality was not enough to conclude if this corresponds to the 1.8-MeV resonance or another resonance expected nearby [28]. Besides, no $J^\pi$ information was obtained. These results limited us to examining in detail the symmetry of the level scheme of $^{12}$O with respect to $^{12}$Be.

In this study, we remeasured the $^{14}$O($p,t$)$^{12}$O reaction accurately to determine the excitation energy and unambiguously assign $J^\pi$. This remeasurement, benefiting from data taken over a longer period of time, achieved higher statistics by almost one order of magnitude than the previous study [37,38], while relying on the same method and nearly the same setup. The missing mass method was used to deduce the excitation energy and the differential cross sections of the reaction. While the detection of recoiling particles is generally challenging in inverse kinematics due to their low energies, the ($p,t$) reaction with a highly negative $Q$ value presents advantageous laboratory-frame kinematics that recoiling tritons direct at small forward angles with an energy of several tens MeV. This unique feature allows us to enhance luminosity by using a thick cryogenic hydrogen target and to optimize the detection efficiency by covering forward angles with an array of MUST2 telescopes [40].

This paper consists of six sections. Sections II, III, IV, and V describe the experimental setup, the analysis, the results, and the discussion, respectively. The paper will conclude with a summary in Sec. VI.

II. EXPERIMENT

The experiment was performed at the LISE beam line [41] of the Grand Accélérateur National d‘ions Lourds (GANIL). A secondary beam of $^{14}$O at 51 MeV/u was produced by the projectile-fragmentation reaction using a $^{16}$O($8^+$) beam. The primary beam with a typical intensity of 500 e nA was accelerated to 90 MeV/u by a pair of cyclotrons, CSS1 and CSS2, and directed to a rotating beryllium target of 4 mm in thickness. The target was tilted at 44° with respect to the beam axis to optimize the beam purity and intensity, measuring 1.1 g/cm² in an average effective thickness. Fragments thus produced were collected and purified by the LISE spectrometer equipped with a 0.5-mm-thick wedge-shaped degrader of beryllium at the dispersive focal plane. To further improve the purity of $^{14}$O, a Wien filter was operated at 150 kV dc at the beginning of the measurement. The recorded data roughly account for one third of the total. The remaining data were taken without using the Wien filter.

The secondary beam was bombarded on a cryogenic hydrogen target [42] at the final focal plane of LISE (Fig. 1).

![FIG. 1. Schematic drawing of the experimental setup.](image-url)
The target was installed in a cylindrical vacuum chamber, referred to as M2C, measuring 1 m both in height and in diameter. The target cell had a circular opening of 1 cm in diameter, which was covered by a pair of Mylar foils having an areal density of 0.8 mg/cm² each. Another pair of the same Mylar foils sandwiched this opening to create volumes for helium gas, which pressurizes the inner volume to ensure homogeneity of the growing slab of solid hydrogen inside. The designed thickness of solid hydrogen was 1 mm, which corresponds to an areal density of 7.1 mg/cm² for the nominal density.

The position and time of beam particles were measured by a pair of multiwire proportional chambers, CATS [43], installed 45 cm and 109 cm upstream of the target, respectively. Each detector has a layer of multiple anode wires sandwiched by two Mylar foils with aluminum electrodes. These cathode electrodes have a square surface of 7 × 7 cm² segmented into 28 strips along the horizontal or the vertical to locate the centroid of image charges, or the impact point of beam particles. To confirm the alignment, calibration measurements were carried out using metallic plates with a series of holes as a position reference. The plate installed 14 cm upstream of each CATS has 1- or 2-mm-diameter holes every 2.5 mm, which serve to collimate the beam into multiple rays and cast a patterned beam image onto the CATS. The holes were reconstructed to the precision of ±0.5 mm. The plates were retracted from the beam line after the calibration run. The time of anode wire signals was recorded by a time-to-analog converter using radio-frequency pulses from SS2 as the stop signal. The recorded time represents the time of flight (TOF) for beam particles to travel the length of the LISE beam line. The detection efficiency was about 90% each. The intensity for beam particles to travel the length of the LISE beam line was about 90% each. The intensity for beam particles to travel the length of the LISE beam line was about 90% each. The intensity for beam particles to travel the length of the LISE beam line was about 90% each.

The plastic telescope consists of a stack of two NE102 plastic scintillators 4 cm apart. A 2-mm-thick ΔE counter is followed by a 10-mm-thick E counter, both having the same surface area of 6 × 6 cm². To read out scintillation light, each scintillator is coupled to a photomultiplier tube (PMT) assembly, model H7415, manufactured by Hamamatsu Photonics. Power supply booster circuits are implemented in these PMT assemblies to limit the attenuation of signals at high counting rates.

Signals from the four MUST2 telescopes were fed to MUFEE front-end boards [44] for pulse shaping and then multiplexed to a VXI-standard MUVI digitizer [44]. The other devices were recorded by 14-bit VXI-standard multipurpose digital converters developed by GANIL. The signal processing of the CATS detectors is described in Ref. [43]. Preamplifier signals of the silicon telescope were fed to an amplifier, model N568B manufactured by CAEN, while PMT output signals of the plastic telescope were recorded directly by a 14-bit digital converter operating in the QDC mode. The trigger generation required at least one DSSD strip to be fired in the whole MUST2 array. The energy thresholds of the DSSDs were set to about 0.5 MeV in the MUFEE boards. To monitor beam and scattered particles without bias from the MUST2 telescopes, logic signals of the CATS and the plastic telescope were added to the trigger after prescaling down by a factor of 10⁶. The data acquisition rate was typically 500 Hz with a live time ratio of 80%.

III. ANALYSIS

Beam particles were identified using the time of anode signals of the downstream CATS with respect to RF pulses (T_{CATS-RF}). Shown in Fig. 2(a) is a scatter plot of the energy loss in the plastic ΔE counter (ΔE_{plast}) against T_{CATS-RF} taken without operating the Wien filter. Prescaled events recorded by the CATS trigger were used. Three distinct clusters correspond to ¹⁴O and isotope contaminants ¹²C and ¹⁵N as labeled. Note that the time of ¹²C is shifted by +76 ns, which corresponds

- ¹²C
- ¹⁴O
- ¹⁵N
to the time interval of RF pulses. This is because $^{12}$C particles that arrive later due to a longer TOF do not share RF pulses of the same bunch as $^{14}$O and $^{13}$N. These clusters are well separated in time as seen in the blank histogram in Fig. 2(b). To select $^{14}$O particles, a gate was set on $T_{\text{CATS-RF}}$ as indicated by the arrows. Figure 2(c) shows the same spectrum, but taken with the Wien filter operated at 150 kV dc. It is seen that the purity of $^{14}$O is improved with the use of the Wien filter.

Trajectories of beam particles were reconstructed using the hit position information from the pair of CATS detectors. The position of beam particles on the target was obtained by extrapolation. The spot size of the $^{14}$O beam was measured to be 4 mm and 1.5 mm RMS in the horizontal and vertical directions, respectively. The spot was 3.5 mm left and 1 mm high relative to the beam axis when viewed from upstream of the target. A 10-mm-diameter cut was set around the beam axis to define the opening of the target and eliminate scattering off the target cell and the heat shield. The rate of $^{14}$O particles accepted by this cut varied from $3 \times 10^4$ to $8 \times 10^4$ pps during the experiment, while the purity stayed at about 65% without the Wien filter and nearly 100% with the filter. This is visualized by the shaded histograms in Figs. 2(b) and 2(c), respectively, that are gated by the circular cut on the beam spot. The total number of $^{14}$O particles used in the analysis was $1.4 \times 10^{10}$, nearly one order of magnitude higher than that of the previous measurement ($1.6 \times 10^9$) [37,38].

The impact point on the MUST2 telescopes was located from the position of hit strips on the front and back sides of a DSSD. The scattering angle in the laboratory frame ($\theta_{\text{lab}}$) was deduced from this information combined with the angle and position of the beam particle at the target. The DSSDs were calibrated by a standard source containing three different $\alpha$ emitters of $^{238}$Pu, $^{241}$Am, and $^{244}$Cm. The typical resolution was about 50 keV FWHM. The energy calibration of CsI was made by analyzing the $E$-$\Delta E$ correlation between the residual energy in CsI and the energy loss in DSSD. CsI energies in analog-to-digital converter (ADC) counts ($E_{\text{CsI}}^{\text{ADC}}$) were calibrated with respect to residual energies calculated by the SRIM code [45] using calibrated DSSD energies ($E_{\text{DSSD}}$) and lengths traveled by ions in DSSD at given impact angles. However, this method based on the $E_{\text{CsI}}^{\text{ADC}}$-$E_{\text{DSSD}}$ correlation loses accuracy when $E_{\text{DSSD}}$ becomes small and causes large uncertainty in predicted residual energies. In the present reaction, the energy deposition in 300-μm-thick silicon is only 1 MeV or less for recoiling tritons with energies higher than 70 MeV. This constituted the major source of error for $E_x$ in the previous measurement (±0.4 MeV) [37,38].

In this analysis, the energy calibration was fine-tuned by reconstructing reaction kinematics. We used the data of the $^{14}$O($p,t$)$^{12}$O reaction and the $^{16}$O($p,t$)$^{14}$O reaction, the latter of which was measured as a reference at 39 MeV/u in separate runs during the beam time. We introduced a one-dimensional adjustment function, $E_{\text{CsI}} = c_0 + c_1 E_{\text{DSSD}}^{(0)}$, where $E_{\text{DSSD}}^{(0)}$ denotes the CsI energy calibrated by the $E_{\text{CsI}}$-$E_{\text{DSSD}}$ correlation. The $c_0$ and $c_1$ parameters were optimized so that the ground state energies of $^{12}$O and $^{14}$O are best reproduced after the reconstruction of the $(p,t)$ reactions. The mass excesses were taken from the latest compilation [26]. The high statistical quality of the present data enabled us to apply this method to each crystal segment. A much improved systematic error of $E_x = \pm 0.1$ MeV was estimated from the reconstructed excitation energies of the $^{14}$O 2$^+_1$ state at 7.7 MeV, that was not taken into account in the tuning of $c_0$ and $c_1$. The total kinetic energy (TKE) was deduced by summing $E_{\text{DSSD}}$ and $E_{\text{CsI}}$ after correcting for the energy deposition in the solid hydrogen target and its window foils. The reaction vertex was assumed at the middle of the target.

FIG. 2. Identification of the secondary beam. (a) Scatter plot of $\Delta E_{\text{plat}}$ vs $T_{\text{CATS-RF}}$. $T_{\text{CATS-RF}}$ spectra (b) without and (c) with turning on the Wien filter. The shaded spectra are gated by a 10-mm-diameter cut to the beam spot on the target. The arrows denote the gate to select $^{14}$O particles.

FIG. 3. Identification of scattered particles by the silicon telescope. (a) Scatter plot of $\Delta E_1$ vs $E_3$ and (b) a magnified view of the enclosed area around the carbon isotopes. The numerical labels denote the mass numbers.
10\textsuperscript{C} particles following the 2\textsuperscript{p} emission decay of 12\textsuperscript{O} were identified by the E-\Delta E method using the silicon and plastic telescopes. The scatter plot of E3 vs \Delta E1 from the silicon telescope is shown in Fig. 3(a). The area around the loci of carbon isotopes is magnified in Fig. 3(b). The locus for 10\textsuperscript{C} is separated by about 4\sigma from those of 9, 11\textsuperscript{C}. The plastic telescope was used to select the atomic number only. During the experiment, both \Delta E and E counters experienced a gain shift and a worsening in energy resolution. This should be due to the damages after irradiation of the secondary beam. The gain shift was corrected for due to the damages after irradiation of the secondary beam. The gain shift was corrected for.

The excitation energy spectrum was analyzed to deduce the resonance energies and widths (\Gamma). Contributions from the breakup reactions of 1\textsuperscript{O} were taken into account. Two types of processes were considered for the breakup to 10\textsuperscript{C}, 2\textsuperscript{p}, and a triton. The first type is that a 1\textsuperscript{O} beam particle directly breaks to the final state without any intermediate states (hereafter referred to as direct breakup). The second proceeds with an intermediate state that consists of 1\textsuperscript{C}, 2\textsuperscript{p}, and 11\textsuperscript{N} which promptly decays into 10\textsuperscript{C} and t (dashed blue line). The inset magnifies the region around the excited state. The center of the Voigt function (solid line) is compared to E\textsubscript{x} = 1.968 MeV (dashed line), the resonance energy reported in Ref. [39]. (d) The same spectrum as (c), but for the fitting function assuming a sequential process for the breakup to 10\textsuperscript{C}, 2\textsuperscript{p}, and t (dashed blue line).

The resulting spectrum is shown by the shaded histogram in Fig. 5(b) after scaling by the total beam counts. The measured background has a flat distribution and much lower magnitude compared to the data with hydrogen. This ensures that the observed resonances originate in interactions with solid hydrogen.

After subtracting the data of the empty target cell, the excitation energy spectrum was analyzed to deduce the resonance energies and widths (\Gamma). Contributions from the breakup reactions of 1\textsuperscript{O} were taken into account. Two types of processes were considered for the breakup to 10\textsuperscript{C}, 2\textsuperscript{p}, and a triton. The first type is that a 1\textsuperscript{O} beam particle directly breaks to the final state without any intermediate states (hereafter referred to as direct breakup). The second proceeds with an intermediate state that consists of 1\textsuperscript{C}, 2\textsuperscript{p}, and 11\textsuperscript{N} which promptly decays into 10\textsuperscript{C} and t (dashed blue line). The inset magnifies the region around the excited state. The center of the Voigt function (solid line) is compared to E\textsubscript{x} = 1.968 MeV (dashed line), the resonance energy reported in Ref. [39]. (d) The same spectrum as (c), but for the fitting function assuming a sequential process for the breakup to 10\textsuperscript{C}, 2\textsuperscript{p}, and t (dashed blue line).

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the phase space. A Voigt function [47], a convolution of a Breit-Wigner function with a Gaussian function, was used to fit an $^{12}$O resonance. The Breit-Wigner function represents the resonance profile and the Gaussian function the detector response. A common Gaussian width was assumed for all resonances at a given angular bin. The $\Gamma$ for the ground state was fixed to 72 keV, the upper limit reported in Ref. [39].

The best fit curves are shown in Figs. 5(c) and 5(d), respectively, for the direct and sequential breakup backgrounds. The sequential breakup background largely reproduces the broad bump dominating the experimental spectrum at higher energies. On the other hand, the fitting with the direct breakup background requires the introduction of at least two additional resonances. The best fit leads to one resonance centered at 4.2 MeV and the other at 7.0 MeV, both with $\Gamma = 2.2$ MeV. The fitting curve reproduces minor rises of the spectrum at these energies, which implies that these resonances could be true. However, the statistical significance of these minor peaks is not as high as the ground state or the state at 1.6 MeV. It is also difficult to identify distinct peaks consistently over angles in the angle-gated spectra shown in Figs. 6. We hence consider that these resonances are only suggestive. Regardless of the breakup types, the $E_x$ of the excited state is 1.61(3) MeV from the fits. The $\Gamma$ is deduced to be 0.83(35) MeV with the direct breakup and 0.38(6) MeV with the sequential breakup. Here the cited errors are statistical.

In our previous study [37,38], the observed resonance is assigned $J^p$ of $0^+$ or $2^+$. While a singlet was assumed given statistical uncertainties, this resonance could be a doublet of a $0^+$ and a $2^+$ state that are expected near 2 MeV from a theoretical prediction [28] as well as the level scheme of $^{12}$Be ($0_2^+$ at 2.25 MeV [29,30] and $2^+$ at 2.10 MeV [32,48]), as pointed out in Ref. [28]. In the present spectra with higher statistics, multiple peaks are still not visible near 1.6 MeV. These possible states could, however, be unresolved within the energy resolution of about 0.4 MeV RMS. We hence carefully deduced $E_x$ and $\Gamma$ by taking the possible doublet into account. In Fig. 7(a), the angular dependence of the yields of the 1.6-MeV peak is shown. This type of presentation, without correcting for experimental conditions, is not common, but is visually helpful to find which $\Delta L$ component governs the experimental yields. Note that an angle-gated spectrum was made every 5° with an angular bin of 10° to take more data points, while retaining enough statistics. In Fig. 7(a), it is seen that the yields of the 1.6-MeV resonance concentrate in a peak at 35°. To find $\Delta L$ primarily responsible for this distribution, we carried out distorted-wave Born approximation (DWBA) calculations with $\Delta L = 0$ and 2. The details of the DWBA calculations will be described later. The calculated differential cross sections were translated into yields by taking into account the solid angle and the detection efficiency calculated by the GEANT4 simulation. The calculated yields were scaled by a common factor to fit the result of $\Delta L = 0$ to the experimental data. As seen in the figure, the calculated distribution for $\Delta L = 0$ nicely reproduces the peak of yields. In contrast, the distribution of $\Delta L = 2$ is too flat to account for the enhancement of yields from 20° to 50°, indicating that its contribution, if any, is minor. A $0^+$ resonance populated with $\Delta L = 0$ is the major component of the peak observed at 1.6 MeV.

If a $2^+$ level is mixed into the 1.6-MeV resonance, its contribution is relatively significant at $\theta_{c.m.} < 20^\circ$ or $\theta_{c.m.} > 50^\circ$, where the yields of the $0_2^+$ state are low. The mixing may shift $E_x$ and $\Gamma$ closer to those of the possible $2^+$ level at these angles. In the $E_x$ vs $\theta_{c.m.}$ plot of Fig. 7(b), the peak energies stay around 1.6 MeV from 20° to 50° and increase up to 2 MeV at smaller and larger angles. A similar trend is also seen in the $\Gamma$ vs $\theta_{c.m.}$ plot of Fig. 7(c) that are around 1 MeV from 25° to 45° and decrease to zero outside. We
are also displayed. The statistical and systematic errors are added. The horizontal bars denote the size of angular backgrounds. The vertical error bars in these figures are statistical. The systematic errors arise from the target thickness estimate (20%) and the detection efficiency simulation (15%). The diffractive patterns of the ground state and the excited 1.6 MeV resonance in the center of mass. The fitting results using the GEANT4 code and the total beam counts obtained from CATS were used. The thickness of the solid hydrogen target was adjusted to reproduce the absolute cross sections of the reference data of the \( ^{16}\text{O}(p,p') \) reaction at 39 MeV/u [37,38]. The detected areal density was 10 mg/cm\(^2\), which is 10% higher than the reported value of 8.85(17) mg/cm\(^2\) [43]. The differential cross sections thus deduced are shown in Figs. 8 for the two types of breakup backgrounds.

FIG. 7. Angular dependence of (a) yields, (b) \( E_x \), and (c) \( \Gamma \) of the 1.6-MeV resonance in the center of mass. The fitting results using the direct (filled circles) and sequential breakup backgrounds (open circles) are shown. The horizontal bars denote the size of angular bins. The adopted values (blue lines) and their errors (shaded areas) are also displayed. The statistical and systematic errors are added.

Reliably to determine the \( E_x \) and \( \Gamma \) of the 0\(^+\) resonance, we take the average of these values over \( \theta_{\text{c.m.}} = 25^\circ \) to \( 50^\circ \). The variation of \( E_x \) is mostly within the systematic error of \( \pm 0.1 \) MeV except for a few angular bins. \( \Gamma \) is particularly sensitive to the level of statistics since the excitation energy resolution of about 1 MeV FWHM is comparable to the peak width. More precise measurements will be necessary to differentiate possibly mixed levels.

The calculated differential cross sections are scaled to the experimental data in Figs. 8. The DWBA angular distributions with \( \Delta L = 0 \) well reproduce the experimental data of both ground and 1.6-MeV states. The scaling factors of \( \Delta L = 0 \) are almost unity for the ground state. This indicates that absolute cross sections of the present reaction calculations are rather reasonable. In the case of the 1.6-MeV state, the scaling factor of \( \Delta L = 0 \) is also nearly unity for the results using.
FIG. 8. Differential cross sections of the $^{14}$O($p,t$)$^{12}$O reaction at 51 MeV/u. The experimental data for (a) the ground $0^+$ state, (b) the excited state at 1.6 MeV, and (c) possible resonances at higher energies are compared to DWBA calculations for $\Delta L = 0$ (solid lines), 1 (dotted lines), and 2 (dashed lines). The results using the direct and sequential breakup backgrounds are denoted by the full circles and full squares, respectively. The previous results [37,38] are also added by the open circles. The vertical and horizontal bars denote the statistical errors and the size of angular bins, respectively.

The sequential breakup background (a factor of 2 for the direct breakup background), while that of $\Delta L = 2$ is more than 5. These results corroborate the $0^+$ assignment to the excited state at 1.6 MeV.

It is interesting that the ground and second $0^+$ states of $^{12}$O carry a similar size of ($p,t$) cross sections. This is in contrast to $^{14}$O, whose ground state has one order of magnitude larger cross sections than the $0^+_1$ state in the $^{16}$O($p,t$) reaction at a proton energy of 54.1 MeV [56]. Assuming that the single-step $2n$ transfer process dominates, cross sections of $\Delta L = 0$ are governed by an overlap between the initial and final state wave functions, and are particularly sensitive to the fraction of the proton ($1p_{1/2})^2$ configuration in the final $0^+$ state. Since $^{14,16}$O are both magic nuclei with a marginal mixing of other intruder configurations in the ground state, the $^{16}$O($p,t$) reaction should favor the ground state of $^{14}$O more than the $0^+_2$ state, which supposedly has a much less ($1p_{1/2})^2$ configuration assuming the symmetry with the $^{14}$C $0^+_2$ state [57]. In the $^{14}$O($p,t$) reaction, in contrast, the disappearance of the shell closure at $Z = 8$ occurs in $^{12}$O [37,38], making the ($1p_{1/2})^2$ component fragmented over the ground and second $0^+$ states to a similar level [28]. This would result in more balanced cross sections from the $^{14}$O ground state. A quantitative discussion requires further reaction analyses based on the coupled reaction channel formalism [58] and fine adjustments of optical potential model parameters using elastic scattering data.

Figure 8(c) shows the differential cross sections deduced from the Voigt functions at 4.2 and 7.0 MeV, respectively, that were obtained from the fits with the direct breakup background. Both angular distributions have a rather smooth and decreasing trend toward larger angles, and the diffractive patterns are not as clear as in the ground and $0^+_2$ states. The cross sections for the 4.2-MeV data slightly rise at 45°, which is in line with the diffractive angle of $\Delta L = 1$. The differential cross sections largely match the DWBA calculation with $\Delta L = 1$ over the angular domain of the measurement. The $J^\pi$ would be $1^-$ if this resonance is true and singlet. The 7.0-MeV data have a shallow dip near 50°, which favors $\Delta L = 0$ over $\Delta L = 1$ and 2. The differential cross sections, however, deviate from the calculation at forward angles with no distinct peak at the predicted diffractive angle around 35°. The $J^\pi$ of this possible resonance is hence not evident.

V. DISCUSSION

The $0^+_2$ state of $^{12}$O is compared to its mirror nucleus $^{12}$Be [30] in Fig. 9. It is seen that the measured excitation energy of the $^{12}$O $0^+_2$ state is lower by 0.63 MeV than that of the $^{12}$Be state. The absolute magnitude of the shift is comparable to a 0.67-MeV shift observed for the $0^+_2$ states between $^{14}$O and $^{12}$Be.

$0^+_2 \quad 2.25 \quad 0.25 \quad 0.19$

$1.62 \quad 1.95 \quad 1.19$

$1^+_0 \quad ^{12}$Be \quad $^{12}$O \quad $^{12}$O-FS \quad $^{12}$O-B

FIG. 9. Comparison of the $0^+_2$ state of $^{12}$O to the mirror state in $^{12}$Be and theoretical predictions of $^{12}$O [28]. The $E_x$ values are given in MeV. The experimental error of the $^{12}$O $0^+_2$ state is denoted by the filled area. The statistical and systematic errors are added. The predictions labeled FS and B are based on the wave functions of Fortune and Sherr [28] and Barker [27], respectively.
while the present result should be considered to be a more significant Coulomb shift given the smaller $Z$ for the $^{12}\text{O}$ and $^{12}\text{Be}$ pair. There are a few theoretical predictions on the $0^+_2$ state of $^{12}\text{O}$ [27,28]. The predicted excitation energies are compared to the data in Fig. 9. While neither prediction perfectly reproduces the data, with Fortune and Sherr’s $E_x$ higher than the experimental value and Barker’s much lower, both are compatible with the data in that the $0^+_2$ state is lowered in $^{12}\text{O}$ with respect to $^{12}\text{Be}$. The two predictions differ in the ratio of valence proton configurations, but they agree that the combined total of the $2s1/2$ and $1p$ configurations amounts to nearly 90%. These orbitals lower the Coulomb energy more readily than other orbitals with higher angular momenta, as their binding energies are near zero. While the ground $0^+$ state is also mixture of $s$ and $p$ configurations, these orbitals extend more into the $0^+_2$ state due to its smaller binding energy. The reduced overlap with the Coulomb field of the core results in a lower Coulomb energy. This qualitatively accounts for the lowering of the excitation energy of the $0^+_2$ state.

To further our understanding, we compare the mirror energy difference (MED) of the $^{12}\text{O} 0^+_2$ state to known mirror $0^+$ states as a function of binding energies. We expect that if the finite-size effects are manifested in the MED, it should introduce a binding energy dependence to the MED, which would otherwise be independent of binding energies. To establish systematics for nuclei with different sizes and charges, we normalize these effects by scaling MED and binding energies. Our MED and its scaling are based on the method proposed in Ref. [49], where even-odd nuclei are studied by the two-body model with one valence proton and a core. In this method, the Fermi energy. The MED of partnering states is defined as the energy difference of the proton and neutron states relative to these thresholds, $ΔE_{\text{MED}} = (E^p_x - S_p) - (E^n_x - S_n)$, where $E^p_x$ and $E^n_x$ denote the excitation energies of the proton- and neutron-rich states, respectively. The MED thus defined is scaled with a nominal Coulomb energy $U$ carried by the valence proton. The resulting $ΔE_{\text{MED}}/U$ ratio, or scaled MED, serves as an indicator of reduction or enhancement of MED. $U$ is given by $6(Z-1)e^2/5R$, assuming a uniformly charged spherical core [49]. $R$ denotes the two-body scaling length, for which the radius formula $1.27(A - 1)^{1/3}$ fm was adopted in Ref. [49]. Here $A$ denotes the mass number of the given nucleus.

In the case of $0^+$ states, the two-body picture may not be appropriate as two nucleons in the same valence orbital equally contribute to the MED. To extend the aforementioned method to $0^+$ states, we adopt the three-body picture in which two valence nucleons are coupled to a core. The MED for two nucleons is defined by replacing $S_p$ and $S_n$ with the two-proton ($S_{pp}$) and two-neutron ($S_{nn}$) separation energies, namely $ΔE_{\text{MED}} = (E^p_x - S_{pp}) - (E^n_x - S_{nn})$. The reference length used for the scaling factor $U$ is also modified. We adopt the hyperradius $R_0$, a widely used length to measure the size of a three-body system. We use the definition that Riisager et al. used in Ref. [12] to scale the radii of $2n$ halo states:

$$ρ_0^2 = \sum_{i<k} \frac{m_im_k}{m_{\text{min}}m_{\text{tot}}} \rho_{ik}^2,$$

where the constituent particles are labeled by the indices $i,k = 0$ to 2. $R_{ik}$ denotes the two-body scaling length between the $i$th and $k$th particles. $m_i$ is the mass of the $i$th particle with $m_{\text{tot}} = m_0 + m_1 + m_2$. $m_{\text{min}}$ is the so-called unit mass that will also be used for the scaling factor of the binding energies. For the three-body model with 2$p$ and the core, the hyperradius reads

$$ρ_0^2 = 2A - 2 \frac{A}{R_{pp}} + \frac{A}{R_{zp}},$$

where $R_{zp}$ denotes the two-body scaling length between the core and a proton and $R_{pp}$ that of the two protons. Here we use the proton mass for $m_{\text{min}}$. We adopt $R_{zp} = r_0(A - 2)^{1/3}$ with $r_0 = 1.27$ fm [49] and $R_{pp} = 2.65$ fm [50]. The choice of these values will be examined later. The scaling factor for two protons is defined as $U_{3BD} = 12(Z - 2)e^2/5\rho_0$. The scaled MED will be referred to as $ΔE_{3BD} = ΔE_{\text{MED}}/U_{3BD}$ hereafter.

FIG. 10. $ΔE_{3BD}$ vs $B_{3BD}$ plots for mirror $0^+$ states. The data of the ground and second $0^+$ states of $^{12}\text{O}$ are denoted by the open and filled circles, respectively. The plots with three different sets of $r_0$ and $R_{zp}$ are shown in panels (a) to (c). Panel (d) magnifies an area with small $B_{3BD}$. For nuclei with $A \leq 18$, mirror states with substantial $2s1/2$ and $1p$ components are denoted by the open squares and solid diamonds, respectively, while the other states are by open triangles. The data include (1) $^{6}\text{Be}$-$^{4}\text{He} 0^+_1$, (2) $^{6}\text{C}$-$^{4}\text{He} 0^+_1$, (3) $^{16}\text{Ne}$-$^{12}\text{C} 0^+_1$, (4) $^{16}\text{Ne}$-$^{12}\text{C} 0^+_2$, (5) $^{14}\text{O}$-$^{12}\text{C} 0^+_1$, (6) $^{10}\text{C}$-$^{12}\text{Be} 0^+_1$, (7) $^{18}\text{Ne}$-$^{16}\text{O} 0^+_1$, (8) $^{18}\text{Ne}$-$^{16}\text{O} 0^+_2$, (9) $^{18}\text{Ne}$-$^{18}\text{O} 0^+_1$, and (10) $^{14}\text{O}$-$^{14}\text{C} 0^+_1$. The labels $α$ and $β$ denote ab initio NCSM calculations [21] for the $^{10}\text{C}$-$^{12}\text{Be} 0^+_1$ pair using the AV8′ interaction [76] and the AV8′ + TM(99) interaction [77], respectively. The dotted and dash-dotted lines are to guide the eyes.
TABLE I. Experimental data and deduced quantities of the mirror $0^+$ states adopted for the $\Delta E_{3BD}$ vs $B_{3BD}$ plots in Fig. 10. The proton- and neutron-rich nuclei of a given mirror pair are listed in the columns of $^A Z_p$ and $^A Z_n$, respectively. The $S_{2p}$ and $S_{2n}$ values are all adopted from the latest compilation [26]. The errors are given in parentheses only when the rounded values are greater than zero.

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<th>$S_{2n}$ (MeV)</th>
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$^a$Present work. The statistical and systematic errors are added in parentheses.

The binding energy of two protons is defined as $B = S_{2p} + 2(Z - 2)\rho_0^2/2 - E_p^0$. The second term is meant to take into account the Coulomb barrier into account using $\rho_0$. The dimensionless binding energy is given by Rissager’s scaling method [12], $B_{3BD} = m_{uls} \beta_{3BD} \hbar^2$.

The $\Delta E_{3BD}$ vs $B_{3BD}$ plot for mirror $0^+$ states with $A = 6$ to 58 is shown in Fig. 10(a). Detailed properties of these states are summarized in Table I. The plot reveals remarkably well-defined correlations between $\Delta E_{3BD}$ and $B_{3BD}$, indicating the validity of the scaling relation to represent MED and binding energies. The plot is roughly divided into two regions above and below $B_{3BD} \approx 5$. The former region with larger binding energies is characterized by a shallow slope of gradually decreasing $\Delta E_{3BD}$. The correlation is nearly linear and most of the data fall within ±5% around the best fit line shown by the dashed line. This linear correlation, however, breaks near $B_{3BD} = 5$. As $B_{3BD}$ nears zero, $\Delta E_{3BD}$ follows a much steeper slope and drops to as low as 60% of the deeply bound data. The $^{12}$O $0^+_1$ state, denoted by the solid circle, has one of the smallest values of $B_{3BD}$. The downward shift of 0.63 MeV for the $0^+_1$ state (Fig. 9) corresponds to 11% of the nominal Coulomb energy $U = 5.7$ MeV. The deduced $\Delta E_{3BD}$ of 0.82 is thus lower by 0.11 compared to that (0.93) obtained for the ground $0^+$ state (open circle), marking the lowest among the data. This lowering is clearly in line with the overall trend of other weakly bound states. While the choice of parameters...
r_0 and R_{pp} is arbitrary to a certain degree, these trends are essentially the same within a reasonable level of changes as shown in Fig. 10(b) with R_{pp} = 2.0 fm and Fig. 10(c) with r_0 = 1.17 fm.

Weakly bound 0^+ states are thus characterized by the drastic lowering of ΔE_{3BD}. To understand its origin, mirror states with substantial 2s1/2 [57,60–63] and 1p components [64–66] are denoted in Fig. 10(d). The other states are either governed by higher L components or deeply-bound. It is seen that the states with lower ΔE_{3BD} with respect to the dashed line mostly involve these components carrying small angular momenta of L = 0 or 1. These components increasingly lose Coulomb energies as B_{3BD} gets smaller, which makes ΔE_{3BD} sharply decline. The gradual evolution of ΔE_{3BD} in the deeply bound states above B_{3BD} = 5 indicates that the Coulomb energy lowers its sensitivity to binding energies and angular momenta. The small ΔE_{3BD} value of the ^{12}\text{O} 0^+_2 state in turn supports the theoretical studies predicting 1p and 2s1/2 configurations for the valence protons [27,28]. While not conclusive from the limited data, the degree of decrease slightly differs between L = 0 and 1 in Fig. 10(d). ΔE_{3BD} of the 2s1/2 states (dotted line) is lowered more than that of the 1p states (dot-dashed line). This is similar to radii of L = 0 neutrons, which show a sharper rise than other L due to the absence of the centrifugal barrier as shown in the calculations of Ref. [75]. The data of both ground and 0^+_1 state of ^{12}\text{O} are in line with the slope of L = 0, suggesting substantial mixing of the 2s1/2 component. This agrees with the mirror states in ^{12}\text{Be}, in which mixing of the 2s1/2 component has been noticed [29–31,33].

It is surprising that such a strict scaling relation of dimensionless ΔE_{3BD} and B_{3BD} universally marks the asymmetry of weakly bound states with various masses. We conclude this discussion by pointing out two possible impacts of this finding. First, the scaling relation of ΔE_{3BD} and B_{3BD} provides a reference to assess to what degree finite binding effects are reproduced in theoretical calculations, almost independently of other effects that influence level energies. It will hence be interesting to see if large-scale shell model or ab initio calculations reproduce this scaling relation. While calculations of separation energies for a pair of mirror 0^+ states are generally lacking, there is one previous ab initio no-core shell model (NCSM) calculation that provides a set of ground state energies of ^{16}\text{C}, ^{18}\text{Be}, and their common core ^{9}\text{Be} [21]. The ΔE_{3BD} and B_{3BD} values were deduced from the predictions made using effective interactions derived from the Argonne V8' (AV8') potential [76] with and without the chiral-symmetry based Tucson-Melbourne TM'99 three-nucleon interaction [77] [β and α in Fig. 10(d), respectively]. While relying on a limited number of harmonic-oscillator bases, both NCSM results with and without the TM'99 interaction well reproduce the experimental data of the ^{10}\text{C}. ^{10}\text{Be} 0^+_1 state, which is down by about 5% to the dashed line and thus indicates a slightly lowered MED. Comparison to future calculations of more unstable mirror nuclei will be interesting.

Another aspect is more related to general properties of low-energy quantum systems. Scaling relations are often thought to be related to universality [12]. Universal phenomena loosely depend on the details of interactions at short distances and thus emerge in various systems at different scales under certain scaling relations. One such example of this long-standing subject in quantum physics is the Efimov three-body state governed by the s-wave scattering length [78]. Debated for nuclear states such as triton, the ^{12}\text{C} Hoyle state [78], or 2n halo states [8,10,79], the first Efimov state has recently been discovered in ultracold atomic gases [80–82]. The work of Ref. [12] points out that the size and binding energy of halo states fulfill a universal scaling rule, which provides a general classification of such states in physics. It is shown that the scaling relation obtained from a few-body model using Woods-Saxon and Gaussian potentials [83] explains experimental data of nuclear halo states as well as theoretical results from Faddeev calculations of other systems such as the ^{4}\text{He} trimmer using realistic potentials of the van der Waals force [84]. It will be interesting to investigate if the scaling of MED is also related to universal aspects of atomic nuclei that have yet to be revealed.

VI. SUMMARY

To study the level scheme of unbound ^{12}\text{O}, we remeasured the ^{14}\text{O}(p,t) reaction at 51 MeV/u using a radioactive beam provided by the LISe fragment separator of GANIL. Statistics were improved by almost one order of magnitude higher than the previous study [37,38]. Recoiling tritons off the cryogenic hydrogen target were detected by an array of MUST2 telescopes. The excitation energy of ^{12}\text{O} and the scattering angle of the reaction were deduced by the missing mass method using the energy and angle of recoiling tritons. A resonance was clearly identified near 1.6 MeV in the resulting excitation energy spectrum. The predicted doublet of a 0^+ and 2^+ state near this energy was carefully dealt with. It was concluded from the angular dependence of the resonance yields that a 0^+ state constitutes the major contribution of the observed resonance. The excitation energy and the natural width of the 0^+ state were deduced to be 1.62 ± 0.03(stat.)±0.10(syst.) MeV and 1.2 ± 0.1(stat.)±0.1(syst.) MeV, respectively, from the data of θ_k.m. = 25° to 45°, where the yields of the 0^+ resonance concentrate. The differential cross sections deduced were well reproduced by DWBA calculations with ΔL = 0, thus confirming the 0^+ assignment. The energies and widths of the resonance were found to shift outside θ_k.m. = 25° to 45°, still suggesting a minor contribution of a 2^+ state. As the possibility that these shifts originate in experimental uncertainties cannot be ruled out, more precise measurements are desirable to conclude or exclude this possible doublet. While other resonances were suggested at 4.2 and 7.0 MeV, they are contingent on the assumption of the direct breakup background.

The excitation energy of the 0^+_2 state in ^{12}\text{O} is by 0.63 MeV lower than the mirror state in neutron-rich ^{12}\text{Be}. This sizable downward shift was discussed in comparison with systematics of other known 0^+ states. To establish the systematics, we introduced a dimensionless mirror energy difference ΔE_{3BD} and binding energy B_{3BD} to normalize the effects of size and charge of nuclei. The ΔE_{3BD} vs B_{3BD} plot indicates a sharp scaling relation between the mirror energy difference and binding energy that is met by various nuclei with different sizes. Weakly bound 0^+ states with L = 0 or 1 valence
nucleons are characterized by lower $\Delta E_{3BD}$ values compared to deeply bound states, which is qualitatively explained by the same mechanism of the Thomas-Ehrman shift. The $\Delta E_{3BD}$ of the $^{12}$O $0^+_2$ state is one of the lowest among the known mirror $0^+$ states, indicating predominant $2s_{1/2}$ or $1p$ configurations for the valence protons as predicted by previous theoretical works [27,28]. The scaling relation of mirror asymmetry is expected to provide a good test for structural calculations in the proximity of the drip lines and to help explore universality inherent in atomic nuclei.

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